Handout 5

(In this handout, for midterm purpose, I will align my notation with the lecture, using tilde to denote per effective labor)

1 Comparative Statics of the *level* of output per worker $\frac{Y}{L}$

Given the production function: $Y = AK_t^{\alpha}(E_t L_t)^{1-\alpha}$, by the construction of intensive form, we define

$$\tilde{y}_t = \frac{Y_t}{E_t L_t} \text{ and } \boxed{y_t = \frac{Y_t}{L_t}} \tag{1}$$

where $\tilde{y}$ is output per effective labor and y is output per worker. From equation 1, we have

$$\boxed{y_t = \tilde{y}_t \cdot E_t} \tag{2}$$

So the growth equation is the following:

$$G(y_t) = G(\tilde{y}_t) + G(E_t) \quad \text{where } G(E_t) = g. \tag{3}$$

Furthermore, recall that at the steady state, the level of $\tilde{y}$ depends on $\tilde{k}$.

The comparative statics of output per labor ($\frac{Y}{L}$) is the following:

- When $n \uparrow$: $\frac{Y}{L}$ will jump to a lower balance growth path (reduction in level).
- When $A \uparrow$: $\frac{Y}{L}$ will jump to a higher balance growth path.
- When $s \uparrow$: $\frac{Y}{L}$ will jump to a higher balance growth path.
- When $g \uparrow$: $\frac{Y}{L}$ is undetermined but will grow to a higher level some time in the future.

  There is no guarantee that during transition in will grow between g and g' and the level will be higher all the time. (need to check!)

2 What steady state are you in?

Notice that when we calculate all the comparative statics, we are in the steady state of

$$\dot{\tilde{k}}_t = 0 \tag{4}$$

This implies that $\tilde{k}_t = \tilde{k}_{t+1} = \tilde{k}_{t+2000}$, so k_t has no growth. Then by the intensive form of production function, we derive that $\tilde{y}_t$ has no growth. Then we can figure our the growth of the different levels. So when we talk about "at the steady state", we mean we want to look at different variables on the balance growth path given that the *capital per effective worker* does not grow.

3 The golden rule of capital accumulation

- When $k_t < k_{\text{gold}}$:
  - Starting point: $s < s_{\text{gold}}$
  - During transition: less consumption
  - At steady state: higher consumption level
- When $k_t > k_{\text{gold}}$:
  - Starting point: $s > s_{\text{gold}}$
  - During transition: more consumption
  - At steady state: higher consumption level
  - The economy is over-saving.
  - The economy is dynamically inefficient.

4 Past Exam

Consider the following equations in the Solow model

$$Y = AK^{\alpha}(EL)^{1-\alpha} \tag{5}$$
$$C = (1-s)Y \tag{6}$$
$$Y = C + I \tag{7}$$
$$\Delta K = I - \delta K \tag{8}$$

The following equations characterize population (L) and labor efficiency (knowledge) (E) growth:

$$L_{t+1} = (1+n)L_t$$
$$Et + 1 = (1+g)E_t$$

(a) Write down the *intensive form* equations (per effective worker) and use "$\sim$ "notation.

(b) Derive steady state quantity of *capital per effective worker*.

(c) In the long-run, what are the growth rates of *output per effective worker* and *output per worker* and *output*?

(d) Illustrate a Solow diagram and show the changes of *capital*, *output*, and *investment per effective worker* in steady state capital if there is a decrease in depreciation (δ) shock.

(e) If δ decreases, what is the change of *consumption per effective worker*?

(f) Suppose a shock permanently reduce the saving rate s *towards* the golden rule rate of saving, what is the change of *consumption per effective worker* in the long run? [continue for (g), (h), (i)]

(g) Furthermore, what is the change of the *growth rate* of *output per worker* $\frac{Y}{L}$ in the long-run?

(h) Furthermore, what is the change of the *level* of *output per worker* $\frac{Y}{L}$ in the long-run?

(i) Construct the transition path of *consumption per effective worker* to the new steady state.

(j) According to the Solow model, if given enough time, do all countries converge to the *same* level of *GDP per worker*?

(k) Prove the *ratio of capital to output* $\frac{K}{Y}$ is stable in the long-run.