Handout 2

1 Logistics

- HW 1 is due on Monday, Oct. 19 at 11 a.m. and HW 2 will be due on Monday, Oct 26 at 11 a.m.
- Handout: http://www.haochehsu.com (Handout can be found at the *Teaching* section)
- Any comments to us, please feel free to use the anonymous *Feedback Survey*.

2 Budget Constraints

Definition 1. *Let $X \subseteq \mathbb{R}^n_+$ be a set of consumption bundles,i.e., the set of all available goods in the economy. A **budget constraint** is a subset of X defined by*

$$\{x \in X : p_1 x_1 + \cdots + p_n x_n = M\}, \tag{1}$$

where M denotes the income, x_i denotes the quantity of good i, and p_i are strictly positive numbers that denotes the price of good i.

In this course, typically $n = 2$, that is, a consumer will face a problem that requires him to choose over 2 alternatives. In this case, we will the budget constraint as the **budget line**, as we can draw a line to represent it on a plane.

Intuitively, the budget means the set of bundles that are *just* affordable to the consumer. A closed-related concept is defined as follows:

Definition 2. *A **budget set** B is a subset of X that is defined by*

$$B(p_1, \ldots, p_n, M) := \{x \in X : p_1 x_1 + \cdots + p_n x_n \leq M\}.$$

Budget set is the set that contains all affordable bundles to a consumer with income M and budget constraint is a special subset of the budget set where the consumer spends all her income. It should be noted that the definition of budget set depends on the prices of the goods and the income level.

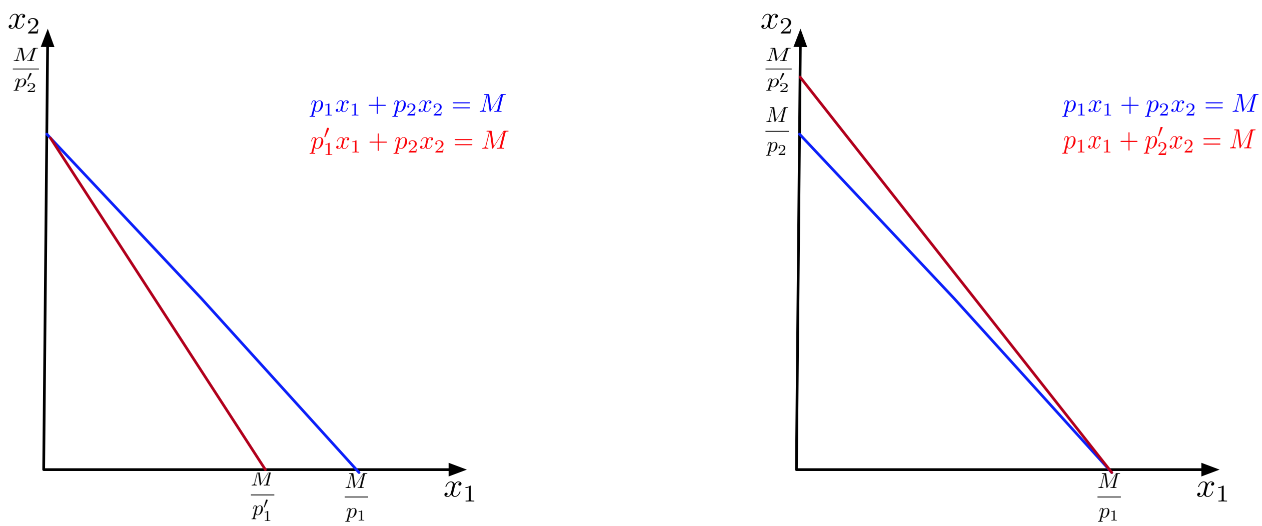
In the case of $n = 2$, one can first identify the budget line and the area inside the budget line and both axis will be the budget set.

2.1 Analysis of Change in Budget Line

In this subsection, we will only consider the case when $n = 2$. Now we can rewrite the (1) as

$$x_2 = \frac{M}{p_2} - \frac{p_1}{p_2} x_1. \tag{2}$$

If the horizontal axis represents x_1 and vertical axis represents x_2, this is a line with negative slope (since $p_1, p_2 > 0$. From (2), it is clear the price ratio p_1/p_2 completely determines the slope of the budget line and M governs the total area of the budget set. That is, if p_1 and p_2 are fixed, the more income one has, the more bundles she can afford.



(a) $p_1' > p_1$

(b) $p_2' < p_2$

Figure 1: The change in budget set given the income M is fixed and one of the prices change.

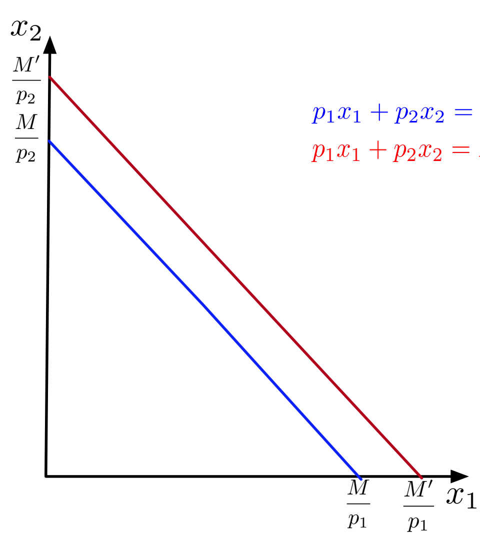


Figure 2: The change in budget set given the prices are fixed and M increases to M'

3 Homework 1 Solutions

1. (True or False) If there are two goods with positive prices and the price of one good is reduced, while income and other prices remain constant, then the size of the budget set is reduced.

   Answer: False

2. (True or False) If all prices are doubled and money income is left the same, the budget set does not change because relative prices do not change.
   Answer: False

3. If all prices double and income triples, then the budget line will become steeper.
   Answer: False

4. (True or False) If there are two goods and the prices of both goods rise, then the budget line must become steeper.
   Answer: False

5. (True or False) A consumer prefers more to less of every good. Her income rises, and the price of one of the goods falls while other prices stay constant. These changes must have made her better off.
   Answer: True

6. If she spends all of her income on uglifruits and breadfruits, Maria can just afford 11 uglifruits and 4 breadfruits per day. She could also use her entire budget to buy 3 uglifruits and 8 breadfruits per day. The price of uglifruits is 6 pesos each. How much is Maria’s income per day?
   Answer: 114 pesos

7. Clara spends her entire budget and consumes 5 units of x and 13 units of y. The price of x is twice the price of y. Her income doubles and the price of y doubles, but the price of x stays the same. If she continues to buy 13 units of y, what is the largest number of units of x that she can afford?
   Answer: 10

8. In year 1, the price of good x was \$3, the price of good y was \$2, and income was \$90. In year 2, the price of x was \$9, the price of good y was \$6, and income was \$90. On a graph with x on the horizontal axis and y on the vertical, the new budget line
   Answer: has the same slope and lies below the original budget line.

9. Isabella thrives on two goods: lemons and tangerines. The cost of lemons is 40 guineas each and the cost of tangerines is 20 guineas each. If her income is 320 guineas, how many lemons can she buy if she spends all of her income on lemons?
   Answer: 8

10. Will spends his entire income on 8 sacks of acorns and 8 crates of butternuts. The price of acorns is 9 dollars per sack and his income is 88 dollars. He can just afford a commodity bundle with A sacks of acorns and B crates of butternuts that satisfies the budget equation

    (a) $9A + 4B = 88$.

    (b) $18A + 4B = 176$.

    (c) $11A + 2B = 88$.

    (d) $9A + 6B = 90$.

(e) None of the above.

Answer: (b)

11. Suppose that the price of good x triples and the price of good y doubles while income remains constant. On a graph where the budget line is drawn with x on the horizontal axis and y on the vertical axis, the new budget line
is steeper than the old one and lies below it.

12. If you spent your entire income, you could afford either 3 units of x and 9 units of y or 9 units of x and 3 units of y. If you spent your entire income on x, how many units of x could you buy?
Answer: 12

13. If you have an income of $\$40$ to spend, commodity 1 costs $\$4$ per unit, and commodity 2 costs $\$8$ per unit, then the equation for your budget line can be written
Answer: $x_1 + 2x_2 = 10$

14. If you could exactly afford either 4 units of x and 24 units of y, or 9 units of x and 4 units of y, then if you spent all of your income on y, how many units of y could you buy?
Answer: 40

15. Murphy used to consume 100 units of X and 50 units of Y when the price of X was $\$2$ and the price of Y was $\$4$. If the price of X rose to $\$4$ and the price of Y rose to $\$9$, how much would Murphy's income have to rise so that he could still afford his original bundle?
Answer: $450

16. This weekend, Martha has time to read 40 pages of economics and 30 pages of sociology. Alternatively, she could read 30 pages of economics and 50 pages of sociology. Which of these equations describes all combinations of pages of economics,E, and sociology, S, that she could read over the weekend?

(a) $E + S = 70$

(b) $E/2 + S = 50$

(c) $2E + S = 110$

(d) $E + S = 80$

Answer: (c)

17. Suppose there are two goods, the prices of both goods are positive, and a consumer’s income is also positive. If the consumer’s income doubles and the price of both goods triple, then
Answer: the slope of the consumer’s budget line does not change but the budget line shifts inward toward the origin.

4 Preferences

Preference relation is used to describe how an agent rank the alternatives. In the later chapter, we will use a concept called *utility function* to describe preference relation. However, the existence of such utility function requires some assumptions of the preference relation. In this class, all the preference relations you encounter will satisfy the following definition:

Definition 3. *A preference relation $\succsim$ on the set of alternatives X is a (binary) relation on X such that*

1. *the preference relation is complete, i.e., for all $x, y \in X$, either $x \succsim y$ or $y \succsim x$ or both cases happen;*
2. *the preference relation is reflexive,i.e., for all $x \in X$, $x \succsim x$.*
3. *the preference relation is reflexive,i.e., for all $x, y, z \in X$, if $x \succsim y$ and $y \succsim z$, then $x \succsim z$.*

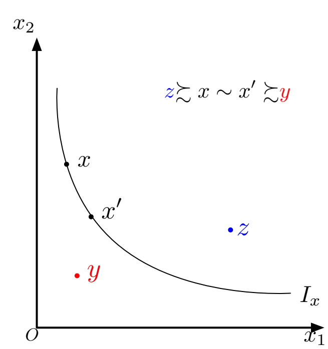
The symbol $\succsim$ can be interpreted as weakly preferred to. For example, $x \succsim y$ can be translated to x is weakly preferred to y. By the completeness assumption, given a preference relation $\succsim$ on X, we can say that

1. x is **strictly preferred** to y, write $x \succ y$, if $x \succsim y$ and $y \not\succsim x$. (translation: x is strictly preferred to y if x is weakly preferred to y and y is not weakly preferred to x).
2. x is **as preferred as** y, write $x \sim y$, if $x \succsim y$ and $y \succsim x$. (translation: x is as preferred as y if x is weakly preferred to y and y is weakly preferred to x).

In most cases in this course, we will be considering the case where an agent has a preference on *two goods*. Here, we made an implicit assumption that the preference is **monotone**, i.e., the more means the better since both commodities are *goods*. Formally, for $n = 2$ case, a preference $\succsim$ on $X = \mathbb{R}^2$ is monotone if for all (x_1, x_2) in X, $(x_1', x_2) \succsim (x_1, x_2)$ whenever $x_1' \geq x_1$ and $(x_1, x_2') \succsim (x_1, x_2)$ whenever $x_2' \geq x_2$.

4.1 Indifference Curves

In the case of two goods, it is convenient to represent the preference relation on a graph by introducing the concept of indifference curves. A indifference curve is a set of bundles whose elements are as preferred as every other elements in the set. Formally, an indifference curve of a bundle x is the set $I_x = \{y \in X | y \sim x\}$. Here is a graphical example of an indifference curve of 2 *goods*.



- ICs do not intersect.
- Marginal rates of substitution (MRS): the slope of a point on the indifference curve.
  - the exchange rates in terms of preference
  - it describes how many units of x_2 one is willing to sacrifice (or be compensated, depending on whether the commodities are goods or not) in order to obtain an additional one unit of x_1.
  - Keep in mind that the willingness to trade here assumes that after trade the consumer will be as happy as before the trade is made.
- Mathematically, if the IC can be described by a function $x_2(x_1)$, then $MRS = \frac{dx_2}{dx_1}(x_1)$.
- If we assume the two commodities are goods, then the slope of the IC is negative, because if you are losing one of the units, you will require some more units of the other goods to be as happy as before.

4.2 Convex Preference

Definition 4. *A preference $\succsim$ is* ***convex*** *if for any $t \in (0,1)$ and for any $x, y \in X$ such that $x \succsim y$, it is true that $tx + (1-t)y \succsim y$.*

Definition 5. *A preference $\succsim$ is* ***strictly convex*** *if for any $t \in (0,1)$ and for any $x, y \in X$ such that $x \succsim y$ and $x \neq y$, it is true that $tx + (1-t)y \succ y$.*

- (strictly) convex preference means the consumer (strictly) prefers diversified bundles over extreme bundles.
- If two commodities are *goods*, there exists a close relationship between MRS and strictly convex preference

  Proposition 1. *Suppose a preference relation is monotone. Then diminishing MRS (in absolute value) is equivalent to strictly convex preference.*

  Question for Fun (that is, these questions are unlikely to be on the exam)

  1. Is there any cases that diminishing MRS will give a convex preference that is not strictly convex?
  2. Without the monotone preference assumption, can you find a strictly convex preference that does not have the diminishing MRS property?

4.3 Exercises

1. Use a coordinate system with ice cream measured on the horizontal axis and chocolate on vertical one to illustrate your preferences by plotting indifference curves if
   (a) You like both ice cream and chocolate a lot.
   (b) You like ice cream, but you hate chocolate

(c) You like ice cream, but you do not care (you are indifferent) about the chocolate

(d) You always eat ice cream and chocolate in the same proportion $(1:1)$

(e) What is the sign of MRS in each of the first three cases?